Gröbner-Shirshov Bases of the Generalized Bruck-Reilly ∗-Extension

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Abstract. In this paper we first define a presentation for the generalized Bruck-Reilly ∗-extension of a monoid and then we work on its Gröbner-Shirshov bases.

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1 Introduction and Preliminaries

In combinatorial group and semigroup theory, for a finitely generated semigroup (monoid), a fundamental problem is to find its presentation with respect to some (irreducible) system of generators and relations and then investigate some algebraic and geometric properties of this semigroup (monoid). In this sense, in [15], the authors obtained a presentation for the Bruck-Reilly extension which was studied previously by Bruck [9], Munn [19] and Reilly [20]. In different manners, this extension is considered as a fundamental construction in the theory of semigroups. Many classes of regular semigroups are characterized by Bruck-Reilly extensions; for instance, any bisimple regular w-semigroup is isomorphic to a Reilly extension of a

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Then the polynomial \((f, g)\) is called the intersection composition of \(f\) and \(g\) with respect to \(w\) if \((f, g)_w = fb - ab\).

(II) Let \(w = \overline{f} = agb\). Then the polynomial \((f, g)_w = f - abg\) is called the elimination of the leading word (ELW) of \(g\) in \(f\).

The first and second compositions given in (I) and (II) are denoted by \(f \wedge g\) and \(f \vee g\), respectively. Moreover, the word \(w\) is called the ambiguity of the composition \((f, g)_w\).

In addition, a composition \((f, g)_w\) is called trivial modulo \((S, w)\), written as \((f, g)_w \equiv 0 \mod (S, w)\), if \((f, g)_w = \sum \alpha_i s_i b_i\) and \(a_1\overline{b}_i < w\), where \(s_i \in S\), \(a_{i}, b_{i} \in X^*\) and \(\alpha_i \in k\). In particular, if \((f, g)_w\) is zero by ELW of polynomials from \(S\), then \((f, g)_w \equiv 0 \mod (S, w)\).

**Definition 1.1.** A monic subset \(S\) of \(k(X)\) is called a Gröbner-Shirshov basis (set) if any composition of polynomials from \(S\) is trivial modulo \(S\) and the ambiguity. In this case, \(S\) is also called a Gröbner-Shirshov basis of the ideal \(\text{Id}(S)\) generated by \(S\) and of the algebra \(k(X; S) = k(X)/\text{Id}(S)\) generated by \(X\) with defining relations \(S\).

The following lemma is due to Shirshov [22] (see also [2] and [3]). It is an analog of the main theorem of Buchberger [10, 11].

**Lemma 1.2.** (Composition-Diamond Lemma) Let \(S \subset k(X)\) be a monic set and let \(<\) be a monomial well ordering of \(X^*\). Then the following conditions are equivalent:

1. \(S\) is a Gröbner-Shirshov basis relative to \(<\).
(2) If \( f \in \text{Id}(S) \), then \( \bar{f} = a\bar{s}b \) for some \( s \in S \) and \( a, b \in X^* \).

(3) \( \text{Irr}(S) = \{ u : u \neq \text{Id}(S), a \neq b X^* \} \) is a \( k \)-base of the algebra \( k\langle X; S \rangle \).

If \( S \) is not a Gröbner-Shirshov basis, then one may construct a Gröbner-Shirshov basis \( R \) of \( \text{Id}(S) \) by adding at each step a non-trivial composition of previous polynomials (and reducing it by the ELW’s of previous polynomials and dividing by the leading coefficient). This process is called the Shirshov algorithm. In general, the Shirshov algorithm is infinite. The reader is referred to [5, 6, 7, 12, 13] for some recent work on Gröbner-Shirshov bases.

For a semigroup \( S \), we denote by \( \mathcal{L} \) and \( \mathcal{R} \) the left and right Green’s congruences on \( S \), respectively. Two elements \( a, b \in S \) are \( \mathcal{L} \)-equivalent \((a\mathcal{L}b)\) if they generate the same left ideal, i.e., if \( S^1a = S^1b \). In a similar way, two elements \( a, b \in S \) are \( \mathcal{R} \)-equivalent \((a\mathcal{R}b)\) if \( aS^1 = bS^1 \). The intersection of \( \mathcal{L} \) and \( \mathcal{R} \) is denoted by \( \mathcal{D} \), while the smallest equivalence containing both \( \mathcal{L} \) and \( \mathcal{R} \) is denoted by \( \mathcal{D}^* \). Additionally, \( a\mathcal{L}^*b \) if for all \( x, y \in S \), \( ax = ay \) if and only if \( bx = by \). Dually, \( a\mathcal{R}^*b \) if for all \( x, y \in S \), \( xa = ya \) if and only if \( xb = yb \). Similarly to the relations \( \mathcal{D} \) and \( \mathcal{H} \), the smallest equivalence containing both \( \mathcal{L}^* \) and \( \mathcal{R}^* \) is denoted by \( \mathcal{D}^* \) and the intersection of them is denoted by \( \mathcal{H}^* \). Evidently, \( \mathcal{L} \subseteq \mathcal{L}^* \), \( \mathcal{R} \subseteq \mathcal{R}^* \), and hence, \( \mathcal{D} \subseteq \mathcal{D}^* \) and \( \mathcal{H} \subseteq \mathcal{H}^* \). \( \mathcal{H}^* \) is an \( \mathcal{H}^* \)-class in the semigroup \( S \).

2 Generalized Bruck-Reilly *-Extension

Let \( T \) be a monoid with \( H_1^* \) (resp., \( H_1 \)) as the \( \mathcal{H} \)-class (resp., \( \mathcal{H} \)-class) containing the identity \( 1_T \) of \( T \). Assume that \( \beta \) and \( \gamma \) are morphisms from \( T \) into \( H_1^* \). Let \( u \) be an element in \( H_1 \) and let \( \lambda_u \) be the inner automorphism of \( H_1^* \) defined by \( x \mapsto uxu^{-1} \) such that \( \gamma \lambda_u = \beta \gamma \). Now we can consider \( S = N_0 \times N_0 \times T \times N_0 \times N_0 \) into a semigroup by defining

\[
(m, n, p, q, r)(m', n', p', q') =
\begin{cases}
(m, n \times p + \max(p, n'), (v'\beta \max(p, n') - p)v'\beta \max(p, n') - n', p' - n' + \max(p, n'), q') & \text{if } q = m', \\
(m, n, v((u - n')(v'\gamma)u'\gamma - m' - 1)\beta u', p, q' - m' + q) & \text{if } q > m', \\
(m - q + m', (u - n')(v'\gamma)u'\gamma - q - 1)\beta u', p' - n', q') & \text{if } q < m',
\end{cases}
\]

where \( \beta'p, \gamma'q \) are interpreted as the identity map of \( T \) and \( u^0 \) is interpreted as the identity \( 1_T \) of \( T \). The monoid \( S = N_0 \times N_0 \times T \times N_0 \times N_0 \) constructed above is called the generalized Bruck-Reilly *-extension of \( T \) determined by the morphisms \( \beta, \gamma \) and the element \( u \). This monoid is denoted by \( S = \text{GBR}^*(T; \beta, \gamma; u) \) and its identity is the element \( (0, 0, 1_T, 0, 0) \).

Lemma 2.1. Suppose that \( X \) is a generating set for the monoid \( T \). Then

\[
\{ (0, 0, x, 0, 0) : x \in X \} \cup (0, 1, 1_T, 0, 1) \cup (1, 0, 1_T, 0, 0) \cup (0, 0, 1_T, 1, 0) \cup (0, 0, 1_T, 0, 1)
\]

is a generating set for the monoid \( S = \text{GBR}^*(T; \beta, \gamma; u) \).

Proof. For \( v, v_1, v_2 \in T \) and \( m_i, n_i, p_i, q_i \in N_0 \) \((1 \leq i \leq 2)\), we can easily show that the proof follows from the equations

\[
(0, 0, v_1, 0, 0)(0, 0, v_2, 0, 0) = (0, 0, v_1v_2, 0, 0),
(0, 0, 1_T, 0, 0)(0, 0, 1_T, 0, 0) = (0, 0, 1_T, 0, 0),
(0, 0, 1_T, 0, 0)(0, 0, 1_T, 0, 0) = (0, 0, 1_T, 0, 0),
(0, 0, 1_T, 0, 0)(0, 0, 1_T, 0, 0) = (0, 0, 1_T, 0, 0),
\]

and so on.
M, any non-empty word
where (1)–(6) onto
as required.
\[ \text{Theorem 2.2. Let } T \text{ be a monoid defined by the presentation } < X; R >, \text{ and let } \beta, \gamma \text{ be morphisms from } T \text{ into } H_1^* \text{. Then the monoid } S = GBR^*(T; \beta, \gamma; u) \text{ is defined by the presentation} \]
\[
\begin{align*}
(X, y, z, a, b : R, \\
yz = 1, ba = 1, \\
xz = x(y\gamma), \\
(2) yx = (x\gamma)y, \\
bx = (x\beta)b, \\
yb = uy, \\
(5) az = zu, \\
bz = zu, \\
(6) xz = z(\gamma).
\end{align*}
\]
where \( x \in X \).

**Proof.** Denote the set \( X \cup \{y, z, a, b\} \) by \( Y \). Let \( \phi : Y^* \rightarrow GBR^*(T; \beta, \gamma; u) \) be the homomorphism defined by
\[
\begin{align*}
x\phi & = (0, 0, x, 0), \\
y\phi & = (0, 0, 1_T, 0, 1), \\
z\phi & = (1, 0, 1_T, 0, 0), \\
a\phi & = (0, 1, 1_T, 0, 0), \\
b\phi & = (0, 0, 1_T, 1, 0).
\end{align*}
\]

By Lemma 2.1, \( \phi \) is an epimorphism. Let us check whether \( GBR^*(T; \beta, \gamma; u) \) satisfies relations (1)–(6). Since the relations \( R \) hold on \( T \), by Lemma 2.1, they also hold on \( GBR^*(T; \beta, \gamma; u) \). The remaining relations (2)–(6) can be checked as follows:
\[
\begin{align*}
(2) : (0, 0, 1_T, 1, 0)(0, 0, 1_T, 0, 0) & = (0, 0, 1_T, 0, 0), \\
(0, 0, 1_T, 0, 1)(0, 0, 1_T, 0, 0) & = (0, 0, 1_T, 0, 0); \\
(3) : (0, 0, 1_T, 0, 1)(0, 0, x, 0, 0) & = (0, 0, x\gamma, 0, 1) = (0, 0, x\gamma, 0, 0)(0, 0, 1_T, 0, 1), \\
(0, 0, x, 0, 0)(1, 0, 1_T, 0, 0) & = (1, 0, x\gamma, 0, 0) = (1, 0, 1_T, 0, 0)(0, 0, x\gamma, 0, 0); \\
(4) : (0, 0, 1_T, 1, 0)(0, 0, x, 0, 0) & = (0, 0, x\beta, 1, 0) = (0, 0, x\beta, 0, 0)(0, 0, 1_T, 1, 0), \\
(0, 0, x, 0, 0)(0, 1, 1_T, 0, 0) & = (0, 1, x\beta, 0, 0) = (0, 1, 1_T, 0, 0)(0, 0, x\beta, 0, 0); \\
(5) : (0, 0, 1_T, 1, 0)(0, 0, u, 0, 1) & = (0, 0, u, 0, 0)(0, 0, 1_T, 0, 1), \\
(0, 0, 1_T, 0, 1)(0, 1, 1_T, 0, 0) & = (0, 0, u^{-1}, 0, 1) = (0, 0, u^{-1}, 0, 0)(0, 0, 1_T, 1, 0); \\
(6) : (0, 0, 1_T, 1, 0)(1, 0, 1_T, 0, 0) & = (1, 0, u, 0, 0) = (1, 0, 1_T, 0, 0)(0, 0, u, 0, 0), \\
(0, 1, 1_T, 0, 0)(0, 0, 1_T, 0, 0) & = (1, 0, u^{-1}, 0, 0) = (1, 0, 1_T, 0, 0)(0, 0, u^{-1}, 0, 0),
\end{align*}
\]
where \( x \in X \). Therefore, \( \phi \) induces an epimorphism \( \overline{\phi} \) from the monoid \( M \) defined by (1)–(6) onto \( GBR^*(T; \beta, \gamma; u) \).

For a given word \( w \), let \( |w| \) denote the length of \( w \). We now show that, in \( M \), any non-empty word \( w \in Y^* \) is equal to a word of the form \( f(m, n, v, p, q) = \ldots \ldots \)
\[ z^m a^n v b^p y^q, \] where \( m, n, p, q \in \mathbb{N}^0 \) and \( v \in X^* \). Here, if \( |w| = 1 \), then we have \( w = x = f(0, 0, x, 0, 0) \) for some \( x \in X \), or \( w = y = f(0, 0, 1, 0, 1) \), or \( w = z = f(1, 0, 1, 0, 0) \), or \( w = a = f(0, 1, 1, 0, 0) \), or \( w = b = f(0, 0, 1, 1, 0) \). Now let us suppose inductively that every word of length less than \( z \) can be reduced to a word of the form \( f(m, n, v, p, q) \) and let \( |w| = k \). Then \( w \) can be written as \( f(m, n, v, p, q)x \) (\( x \in X \)), or \( f(m, n, v, p, q)y \), or \( f(m, n, v, p, q)a \), or \( f(m, n, v, p, q)b \), or \( f(m, n, v, p, q)z \). Now

\[
\begin{align*}
 f(m, n, v, p, q)x &= z^m a^n v b^p y^q x = z^m a^n v (x(m+1))(b^p y^q) = f(m, n, v(x(m+1))(b^p y^q), p, q); \\
 f(m, n, v, p, q)y &= z^m a^n v b^p y^q y = z^m a^n v b^p y^{q+1} = f(m, n, v, p, q+1); \\
 f(m, n, v, p, q)a &= z^m a^n v b^p y^q a = \begin{cases} 
 f(m, n, v((u^{-1})q^{-1})(b^p, p, q) \text{ if } q \geq 1, \\
 f(m, n, v, p, p-1, 0) \text{ if } q = 0, p \geq 1, \\
 f(m, n, v, p+1, 0) \text{ if } q = p = 0; 
\end{cases} \\
 f(m, n, v, p, q)b &= z^m a^n v b^p y^q b = \begin{cases} 
 f(m, n, v((u^{-1})q^{-1})(b^p, p, q) \text{ if } q \geq 1, \\
 f(m, n, v, p, p+1, 0) \text{ if } q = 0; 
\end{cases} \\
 f(m, n, v, p, q)z &= z^m a^n v b^p y^q z = \begin{cases} 
 f(m, n, v, p, q-1) \text{ if } q \geq 1, \\
 f(m, n+1, 0, u^{-1}(v(m+1)), 0, 0, 0) \text{ if } q = 0. 
\end{cases}
\end{align*}
\]

So the inductive step is complete.

Finally, if \( f(m_1, n_1, v_1, p_1, q_1) \phi = f(m_2, n_2, v_2, p_2, q_2) \phi \), then

\[
(z^m a^n v b^p y^q) \phi = (z^m a^n v b^p y^q) \phi,
\]

and so \( (m_1, n_1, v_1, p_1, q_1) = (m_2, n_2, v_2, p_2, q_2) \), where \( v_1, v_2 \in X^* \) and \( m_i, n_i, p_i, q_i \in \mathbb{N}^0 \) \( (1 \leq i \leq 2) \). Hence, \( v_1 = v_2 \) in \( T \) and \( m_1 = m_2, n_1 = n_2, p_1 = p_2 \) and \( q_1 = q_2 \). Since the presentation of \( GBR^*(T; \beta, \gamma; u) \) contains \( R \), we deduce that \( v_1 = v_2 \) holds in \( M \), and thus \( z^m a^n v b^p y^q \) is injective, as required.

**Corollary 2.3.** Let \( v \) be an arbitrary word in \( X^* \). The relations

\[
y^m v = (v^{m+1})y^m, \quad v z^m = z^m (v^{-1} m), \quad b^p v = (v^{p+1})b^p, \quad va^n = a^n (v^p b^p), \\
y^m b^p = (v^{m+1} y^m), \quad y^n a^n = (u^{m+1} (m+1)) y^m, \\
8 b^p z^m = z^m (u^{m+1} y^m), \quad a^n z^m = z^m (u^{m+1} y^m)
\]

hold in \( GBR^*(T; \beta, \gamma; u) \) for all \( m, n \in \mathbb{N}^0 \). As a consequence, every word \( w \in (X \cup \{y, z, a, b\})^* \) is equal in \( GBR^*(T; \beta, \gamma; u) \) to a word of the form \( z^m a^n v b^p y^q \) for some \( v \in X^* \) and \( m, n, p, q \in \mathbb{N}^0 \).

Now we discuss Gröbner-Shirshov bases of the generalized Bruck-Reilly \(*\)-extension. Let \( T \) be the monoid defined by the presentation \( \langle X; R \rangle \). By Theorem 2.2, the generalized Bruck-Reilly \(*\)-extension \( GBR^*(T; \beta, \gamma; u) \) is defined by a presentation

\[
\{X, y, z, a, b : R, \quad y z = 1, \quad ba = 1, \quad y x = (x(m+1)), \quad x z = z(x(m+1)), \quad bx = (x(m+1))b, \\
xa = a(x(m+1)), \quad yb = yu, \quad ya = u^{-1} y, \quad bx = zu, \quad az = z(u^{-1}) \},
\]

where \( x \in X \). We note that throughout this section we assume that \( |x(m+1)| = |x(m+1)| = |u| = 1 \). Hence, we get \( x(m+1), x(m+1), u \in X \). Now let us order the set \( \{X \cup \{y, z, a, b\}\}^* \).
lexicographically by using \( y > b > x > a > z \) \((x \in X)\), and then consider the polynomials

\begin{align*}
(i) & \ r - v, \quad (ii) \ yz - 1, \quad (iii) \ ba - 1, \quad (iv) \ yx - (x\gamma) y, \\
(v) & \ bx - (x\beta) b, \quad (vi) \ xz - z(x\gamma), \quad (vii) \ xa - a(x\beta), \quad (viii) \ gb - uy, \\
(ix) & \ ya - u^{-1} y, \quad (x) \ az - zu^{-1}, \quad (xi) \ bz - zu,
\end{align*}

where \( r = v \in R \). Also, we can assume that \( R \) is a minimal Gröbner-Shirshov basis for the monoid \( T \) in a sense that the leading monomials are not contained in each other as subwords, in particular, they are pairwise different. Now let us label some words with the notations

\[ \overline{r}: \ \text{word without the last letter of the word} \ r, \]

\[ \overline{r}: \ \text{word without the first letter of the word} \ r. \]

\textbf{Theorem 2.4.} A Gröbner-Shirshov basis for \( GBR^*(T; \beta, \gamma; u) \) presented by (7) consists of polynomials (i)–(xi).

\textbf{Proof.} We need to prove that all compositions of polynomials (i)–(xi) are trivial. To do that, firstly, let us consider intersection compositions of (i) with (vi) and (vii).

The ambiguities are the following:

\begin{align*}
(i) \cap (vi) & \ : w = \overline{r}xz, \\
(f, g)_w & = (r - v)z - \overline{r}(xz - z(x\gamma)) = rz - vz - rz + \overline{r}z(x\gamma) = \overline{r}z(x\gamma) - vz = z(\overline{r}\gamma)(x\gamma) - z(v\gamma) = z(r\gamma) - z(v\gamma) \equiv 0; \\
(i) \cap (vii) & \ : w = \overline{r}xa, \\
(f, g)_w & = (r - v)a - \overline{r}(xa - a(x\beta)) = ra - va - ra + \overline{r}a(x\beta) = \overline{r}a(x\beta) - va = a(\overline{r}\beta)(x\beta) - a(v\beta) = a(r\beta) - a(v\beta) \equiv 0.
\end{align*}

Next we proceed with intersection composition of (iii) with (x), and we have

\begin{align*}
(iii) \cap (x) & \ : w = baz, \\
(f, g)_w & = (ba - 1)z - b(az - zu^{-1}) = baz - z - baz + bzu^{-1} = bzu^{-1} - z = zu^{-1} - z \equiv 0.
\end{align*}

By intersection compositions of (iv) with (i), (vi) and (vii), we get the following ambiguities:

\begin{align*}
(iv) \cap (i) & \ : w = yxz, \\
(f, g)_w & = (yx - (x\gamma)y)z - y(r - v) = yxz - (x\gamma)yz - yr + yv = yv - (x\gamma)yz = (v\gamma)y - (x\gamma)yz = (v\gamma)y - (r\gamma)y \equiv 0; \\
(iv) \cap (vi) & \ : w = yxxa, \\
(f, g)_w & = (yx - (x\gamma)y)z - y(xz - z(x\gamma)) = yxz - (x\gamma)yz - yxz + yz(x\gamma) = -(x\gamma)yz + yz(x\gamma) \equiv 0; \\
(iv) \cap (vii) & \ : w = yxa, \\
(f, g)_w & = (yx - (x\gamma)y)a - y(xa - a(x\beta)) = yxa - (x\gamma)ya - yxa + ya(x\beta) = ya(x\beta) - (x\gamma)ya = u^{-1}y(x\beta) - (x\gamma)u^{-1}y = u^{-1}((x\beta)\gamma)y - (x\gamma)u^{-1}y,
\end{align*}
and since $\gamma\lambda_u = \beta\gamma$, we get $u^{-1}(x\beta)\gamma y - (x\gamma)u^{-1}y = u^{-1}(x\beta)\gamma u - (x\gamma)u^{-1}y = u^{-1}u(x\gamma)u^{-1}y - (x\gamma)u^{-1}y \equiv 0$.

Now we consider intersection compositions of (v) with (i), (vi) and (vii):

$$(v) \land (i) : w = bxz,$$

$$(f, g)_w = (bx - (x\beta)b)x - b(r - v) = bxz - (x\beta)br - br + bv = bv - (x\beta)br \equiv 0;$$

$$(v) \land (vi) : w = bxz,$$

$$(f, g)_w = (bx - (x\beta)b)z - b(xz - z(x\gamma)) = bxz - (x\beta)bz - bxz + bz(x\gamma) = bz(x\gamma) - (x\beta)bz = zu(x\gamma) - (x\beta)zu = zu(x\gamma) - z((x\beta)\gamma)u,$$

and since $\gamma\lambda_u = \beta\gamma$, we obtain $zu(x\gamma) - z((x\beta)\gamma)u = zu(x\gamma) - z((x\gamma)\lambda_u)u = zu(x\gamma) - zu(x\gamma)u^{-1}u \equiv 0$.

$$(v) \land (vii) : w = bxz,$$

$$(f, g)_w = (bx - (x\beta)b)a - b(xa - a(x\beta)) = bxa - (x\beta)ba - bxa + ba(x\beta) = ba(x\beta)ba \equiv 0.$$

Next we proceed with intersection compositions of (vii) with (x), (viii) with (iii), (v) and (xi), and (ix) with (x), hence we get

$$(vii) \land (x) : w = xaz,$$

$$(f, g)_w = (xa - a(x\beta))z - x(az - zu^{-1}) = xaz - a(x\beta)z - xaz + xzu^{-1} = xzu^{-1} - a(x\beta)z = z(x\gamma)u^{-1} - az((x\beta)\gamma),$$

and since $\gamma\lambda_u = \beta\gamma$, we get $z(x\gamma)u^{-1} - az((x\beta)\gamma) = z(x\gamma)u^{-1} - az((x\gamma)\lambda_u) = z(x\gamma)u^{-1} - azu(x\gamma)u^{-1} = z(x\gamma)u^{-1} - zu(x\gamma)u^{-1}u \equiv 0$.

$$(viii) \land (iii) : w = yba,$$

$$(f, g)_w = (yb - uy)a - y(ba - 1) = yba - uy - yba + y = y - uy = y - uu^{-1}y \equiv 0;$$

$$(vii) \land (v) : w = bxz,$$

$$(f, g)_w = (yb - uy)x - y(bx - (x\beta)b) = ybx - uyx - ybx + y(x\beta)b = y(x\beta)b - uyx = ((x\beta)\gamma)y - uy(x\gamma)y,$$

and we get $((x\beta)\gamma)y - uy(x\gamma)y = ((x\gamma)\lambda_u)uy - u(x\gamma)y \equiv 0$ since $\gamma\lambda_u = \beta\gamma$.

$$(vii) \land (xi) : w = ybz,$$

$$(f, g)_w = (yb - uy)z - y(bz - zu) = ybz - uy - ybz + yzu = yzu - uy - yzu \equiv 0;$$

$$(ix) \land (x) : w = yaz,$$

$$(f, g)_w = (ya - u^{-1}y)z - y(az - zu^{-1}) = yaz - u^{-1}y - yaz + yzu^{-1} = yzu^{-1} - y^{-1}y \equiv 0.$$
Finally, it remains to check compositions of inclusion of polynomials (i)–(xi). But it is clear that there are no compositions of this type. Hence, the result follows.

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References

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